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Development of an energy detection algorithm in MATLAB when Noise power is perfectly known

Nwukor, F. N. and Omorogiuwa, O. S.
Petroleum Training Institute, Effurun, Delta State, Nigeria
Email: francesnkem@gmail.com
Department of Electrical and Electronics Engineering University of Benin, Benin City, Nigeria
Email: sam@uniben.edu

Abstract
This paper developed an energy detection algorithm using MATLAB to determine the availability of a spectrum by testing the spectrum for false alarms using the energy detector. The research was carried out on a system whose Noise Power value was known before the detection and was fixed.
Additive White Gaussian Noise distribution test Statistics was used to analyze when Noise power is perfectly known, and develop an energy detection algorithm in MATLAB to evaluate probability of detection, probability of false alarm, and signal to noise ratio.
The design is shown in the simulated probabilities while the derivative is theoretical. This approach shows the correlation between the theoretical and simulated values of probability of detection for the given range of probability of false alarm. The research in this paper has shown that there was a high correlation between simulated and theoretical values of the ROC plots for different SNRs. The higher the SNR value for the energy detector, the better the performance and the lower the value, the reverse for the energy detector. This value turned out to be approximately 95% for the case of known noise power, using a bit of heuristics to determine the fairly accurate noise floor. However, considering the 5dB SNR, it clearly distinguishes between the heuristics H0 and H1. So, the expectation was that the detector would have 100% detection for any false alarm probability.

Keywords: ***Energy detection, Noise, Spectrum, detection, false alarm.***

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I. Introduction

In recent times, new communication technologies evolve every day and these technologies have a need of use for the RF spectrum for their functionality. However, the RF spectrum seems already occupied because of communication governing bodies [1]. These spectrums are allocated in sections to different defined uses. As a result, it has affected the use of spectrum in the area where a large amount of spectrum is needed, which brings about spectrum scarcity in some devices; this scarcity has led to poor communications that cause loss of data.

It has been reported that cognitive radio's can solve the problem of spectrum underutilization and artificial scarcity because of its ability to be used in the form of communication, but most do not interfere with other users [1]. Most researchers had used CR to investigate, that there are spectrum holes and reuse of the spectrum hole through the energy detection method [1]

The energy detection method involves an assumption between samples or two-state processes that contain noise power and higher noise signal power [2]. To determine high noise or low noise the received signal is sampled [2]. When the energy detection algorithm is executed on any spectrum, the threshold value should theoretically depend on the radio class of the station that may or may not be occupying a particular radio channel.

The best solution to these problems is Software Defined Radio [3]. GNU, a recursive acronym for "GNU's Not Unix", provides a cognitive radio network [1, 4].

Energy detection involves observing the spectral energy across the visible spectrum and using a threshold to decide if a signal is there at each frequency. Energy detection typically occupies the use of the power spectral density of the input signal as a detection statistic.

II. Theoretical Background

When the signal has passed through the band-pass filter at a given bandwidth (W) the signal received is represented in Eq. (1) [5].

$r(t) = \{v(t)\ s(t) + v(t)\ H_o\ H_i$ (1)

From Eq. (1) is shown that $v(t)$ is the Gaussian noise, $s(t)$ is the received signal of the bandpass. H_o this is when the signal is not available and H_i is the signal available.

To determine the noise in the paper the noise power is denoted in [5]:

$\sigma^2 = E\{v^2(t)\} = N_o W$ (2)

where E is the epsilon.

The signal that will be analyzed by the energy detector during a period of time will be compared to a threshold as denoted in Eq. (3) [6].

$\tilde{T} = \int_0^T r(t)^2 dt \gtrless \xi$ (3)

When the sampling has been done at time (t_i), the low-pass equivalent of discrete-time is represented in Eq. (2.1) as shown in Eq. (4) [5].

$y_i = \{n_i\ x_i + n_i\ H_o\ H_i$ (4)

where $y_i = y(t_i)$, $x_i = x(t_i)$ and $n_i = n(t_i)$ and $y(t)$, $x(t)$ and $n(t)$ are the equivalent low-pass representations of $r(t)$, $s(t)$ and $v(t)$ correspondingly.

The discrete form of Equation (3) when normalized dimensionless in metric form by dividing by $N_o/2$ becomes as shown in Eq. (5) [6].

$\frac{1}{\sigma^2}\sum_{i=1}^{N} |y_i|^2$ (5)

To improve the results correctness, it is when $N \approx 2TW$ it increases, this is in a scenario $TW \gg 1$, this will make time-discrete and time-continuous of the Energy Detection difference between them negligible when compared to Eq. (3). The metric in Eq. (5) is called the test statistics in time-discrete performance, the Energy Detection (ED) becomes Eq. (6) [5].

$V \triangleq \frac{1}{\sigma^2}\sum_{i=1}^{N} |y_i|^2 H_i \gtrless H_o\ \xi$ (6)

The threshold, ξ is set based on the noise power, σ^2 for a given T, W and a required P_{FA}. In a practical circumstance, σ^2 it is estimated as $\hat{\sigma}^2$ [7, 8] such that the ED statistics becomes Eq. (7) [5].

$\hat{V} = \frac{1}{\hat{\sigma}^2}\sum_{i=1}^{N} |y_i|^2$ (7)

2.1. Energy Detector

Considering a signal $s(t)$, which is unknown but deterministic then, the generic sample y_i is represented as Gaussian distributed, i.e. y_i with a mean of zero and variance of $2\sigma^2$ written as $y_i \sim C\aleph(0, 2\sigma^2)$ which $C\aleph(.)$ indicates a circularly-symmetric complex Gaussian random variable. This is the condition for hypothesis H_o while that of $H_1$is $y_i \sim C\aleph(x_i, 2\sigma^2)$. It can be observed that for hypothesis H_o, the distribution is seen as a central chi-squared distribution while that of H_1 is seen as a non-central chi-squared distribution with non-centrality parameter $\lambda = \frac{\sum_{i=1}^{N} |x_i|^2}{\sigma^2}$.

2.2. Distribution of Test Statistics

As already stated for H_o the distribution is a central chi-squared distribution which is given as Eq. (8) [9];

$f_{\tilde{T}}(x) = \frac{x^{N-1} e^{-x/2\sigma^2}}{(2\sigma^2)^N \Gamma(N)}$ (8)

Where $\Gamma(n) = \int_0^\infty t^{n-1} e^{-t} dt$ is the gamma function? Thus, the probability of false alarm (P_{FA}) can be derived as Eq. (9) [5]:

$P_{FA} = \int_x^\infty f_{\tilde{T}}(x) dt = \frac{\Gamma(N, \frac{\xi}{2\sigma^2})}{\Gamma(N)}$ (9)

Similarly, for the hypotheses H_1, the probability of detection P_D of the non-central chi-squared distribution is shown in Eq. (10):

$P_D = Q_N(\sqrt{N}, \sqrt{\xi})$ (10)

Where, $Q_k(\alpha,\beta) = \int_{\beta}^{\infty} x\left(\frac{x}{\alpha}\right)^{k-1} e^{-x^2+\frac{\alpha^2}{2}} I_k(\alpha x)dx$ is the generalized Marcom's Q function of the order k.

III. Methodology

3.1. Energy Detection Algorithm in MATLAB

Considering a signal $s(t)$, which is unknown but deterministic, then, the generic sample y_i is Gaussian distributed. This is the condition for hypothesis H_o while that of H_1 is $y_i \sim \mathbb{C}\aleph(x_i, 2\sigma^2)$, the analysis of this test statistic has been described in (Eq. (7) to Eq. (9)).

Having specified certain theoretical equations (Eq. (1) to (10)), it will be better to improve the present design of the energy detector though its performance is tested with P_{FA} and P_D as well as the accuracy of measurement with N.

For this design to be very appreciated, it must avoid interfering with the primary users and other user in the link. And the desired probability of detection must be greater than desired probability of detection i.e ($P_D > P_D^{DES}$) and with probability of false alarm to be small. Sometimes, the maximum desired P_{FA} indicated is P_{FA}^{DES} (desired probability of false alarm), as $P_{FA} < P_D^{DES}$.

Also, P_D is a function of sampling time, N, of the threshold ξ and of the Signal to Noise Ratio (SNR) while P_{FA} is a function of N and ξ but not of SNR, i.e.,

$$P_{FA} = f(\xi, N) \quad (11)$$

$$P_D = d(SNR, \xi, N) \quad (12)$$

Where $f(.,.)$ and $d(.,.,.)$ are functions of multiple variables. Thus, a given detector would have to be designed towards a particular performance that is derived as a (P_{FA}^{DES}) pair. The threshold ξ^* and the minimum SNR, SNR_{min}, satisfying the specifications by inverting P_D and P_{FA}.

3.2. Designing an Energy Detection fulfilling both P_{FA} and P_D

In this case of design, the energy detection (ED) fulfills both P_{FA} and P_D and with the improvement of desired P_D and P_{FA}.

$$P_{FA} < P_{FA}^{DES} \quad (13)$$

$$P_D > P_D^{DES} \quad (14)$$

The threshold and the minimum required SNR fulfilling Eq. (13) and Eq. (14) are given respectively by;

$$\xi^* = f^{-1}\left(P_{FA}^{DES}; N\right) \quad (15)$$

$$SNR_{min} = d^{-1}\left(P_D^{DES}; \xi^*, N\right) \quad (16)$$

3.3. Designing an Energy Detection fulfilling the constraint $P_D > P_D^{DES}$ for SNR $> SNR_{min}$

In this case of design, the ED fulfills the constraint $P_D > P_D^{DES}$ for SNR $> SNR_{min}$ and with improvement. Here the threshold is

$$\xi^* = d^{-1}\left(P_D^{DES}; SNR_{min}, N\right) \quad (17)$$

Resulting in

$$P_{FA} = f(\xi^*, N) \quad (18)$$

Clearly, increasing N will decrease P_{FA}. Given a desired pair of probabilities, the threshold and minimum SNR can be expressed as Eq. (19) and Eq. (20) respectively by considering equations (15, 7 and 8)

$$\xi^* = f^{-1}(P_{FA}^{DES}; N) = \left(Q^{-1}(P_{FA}) + \sqrt{N}\right)\sqrt{N}\sigma^2 \quad (19)$$

Equation (19) was the threshold computation for energy detection. It should be noted that the calculated threshold is obtained from the case of a perfectly known noise power. Also, for SNR_{min} from the knowledge of the noise power from equations (8) and (9), the SNR min can be articulated as shown in Eq. (20).

$$SNR_{min} = \frac{Inv\Gamma(N, P_{FA}^{DES})}{Inv\Gamma(N, P_D^{DES})} - 1 \quad (20)$$

The inverse gamma regularized function is shown as, $Inv\tilde{\Gamma}$ (.,.), (if $z = (a, w)$, then $w = (a, z)$). Further analysis was done to simplify the expression for variance and phase, which is the SNR wall.

$$SNR_{\min} = SNR_{wall} = \frac{1 - Q^{-1}\left(P_D^{DES}\right)\sqrt{\phi}}{1 - Q^{-1}\left(P_{FA}^{DES}\right)\sqrt{\phi}} - 1 \qquad (21)$$

Where $\phi = Var\left(\frac{\sigma^2}{\hat{\sigma}^2}\right)$ and $Var(\ )$ is the variance. This expression can be approximate as $\phi = \sqrt{\frac{N+M}{NM}}$.

For the design the sample number (N) is an important parameter to achieve the requirements on detection and false alarm probabilities of the sample. For a given false alarm probability, P_{FA}^{DES} and detection probability P_D^{DES}, the minimum required number of samples can be given as a function of SNR. By eliminating from both P_{FA} in equation (8) and P_D in equation (9) the equation is shown in (22).

$$N = \left[Q^{-1}\left(P_{FA}^{DES}\right) - Q^{-1}\left(P_D^{DES}\right)\sqrt{2SNR+1}\right]^2 \times SNR^2 \qquad (22)$$

Equation (22) should be noted that the function of the threshold due to the monotonically decreasing property of function $Q^{-1}(.)$

From the analysis when N is increased the signal can be detected even in a very low SNR region if the noise power is perfectly known.

3.4. Implementation in MATLAB.

The calculation was done for predetermined values and computed values.

i. Predetermined values: The values of the probabilities P_D and P_{FA} are chosen as 0.99 and 0.01 respectively in order to satisfy IEEE 802.22 WRAN standards. The maximum sample rate when using RTL-SDR is 3.2MHz, and can be used in analysis. But it is best to use a recommended sample rate of 3MHz in order to reduce the number of dropped frames/samples as shown in Figure 1-4.

However, the choice of sample rate was determined by the number of samples to be considered, and similarly, the bin width is dependent on the bandwidth of the signal being considered (in our case it is the primary spectrum signal which has a bandwidth of 200kHz). In turn, the bin width depends on the sample rate and the number of samples/FFT size.

Ordinarily, the number of samples and the FFT size is not tied together but due to the use of the RTL-SDR for spectrum sweeping when considering the sample rate, considerations have to be made to the FFT size and bin width as well as several samples to satisfy the recommended conditions for a standard design.

ii. Computed values: the bandwidth of the FM signal is 200kHz, and the maximum bin width for FFT is 100kHz. Several samples/FFT size should be greater than the minimum number of samples for an SNR of -10dB and has an upper limit due to an FFT size of no more than the sample rate/bin width.

The threshold is computed using Eq. (22) but with σ^2 replaced with estimated noise power $\hat{\sigma}^2$ which was computed with the same number of samples as N (i.e., N = M). Test statistics equation is replaced with Eq. (19) for the sample number of noise only samples M for SNR wall of -10dB.

The code was done with MATLAB, for the energy detector of a signal mixed with noise (AWGN) at the SNR wall of -20dB according to IEEE 802.22 standard while measuring the performance through the probabilities, P_{FA} and P_D of the detector as shown in Figure 1-4.

The energy detector used the threshold formula of Eq. (19) for several samples N of 2^{18} which is higher than the minimum number of samples according to Eq. (22) given as 218635.1766. Due to a large number of samples, there was a high amount of sensing time (about 30 minutes with a Personal Computer).

The code implements P_D computation at a given P_{FA} for 1000 Monte Carlo iterations, such that the value of P_D is the ratio of the number of times the energy detector test statistics exceeds the threshold.

IV. Results

The results in Figures 1 to 3 show the ROCs plots for three different SNR values where each plot is a comparison of the ROC between the derived theoretical and simulated probabilities. The design of when noise power is perfectly known is shown in the simulated probabilities while the derived is theoretical. This approach of analysis is taken to show the correlation of theoretical and simulated values of probability of detection for the given range of probability of false alarm.

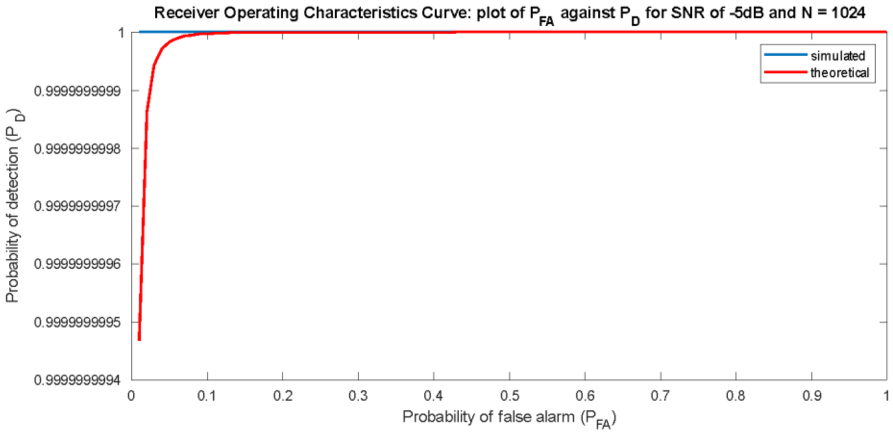

Figure 1: Receiver Operating Characteristics for SNR of -5dB

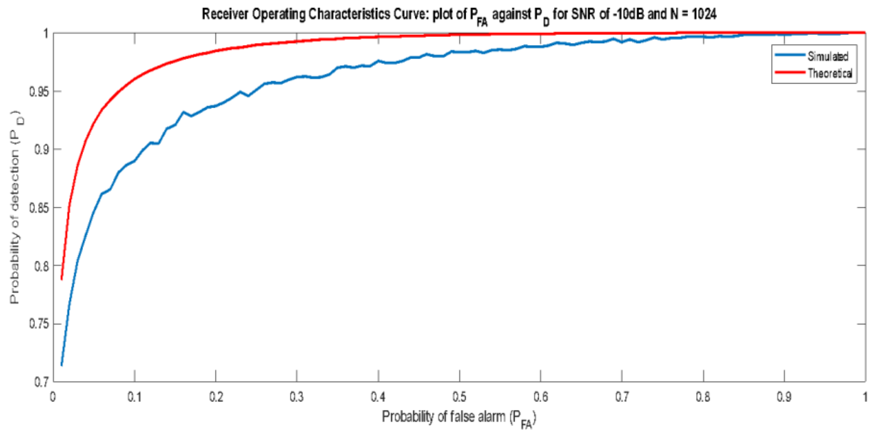

Figure 2: Receiver Operating Characteristics for SNR of -15dB

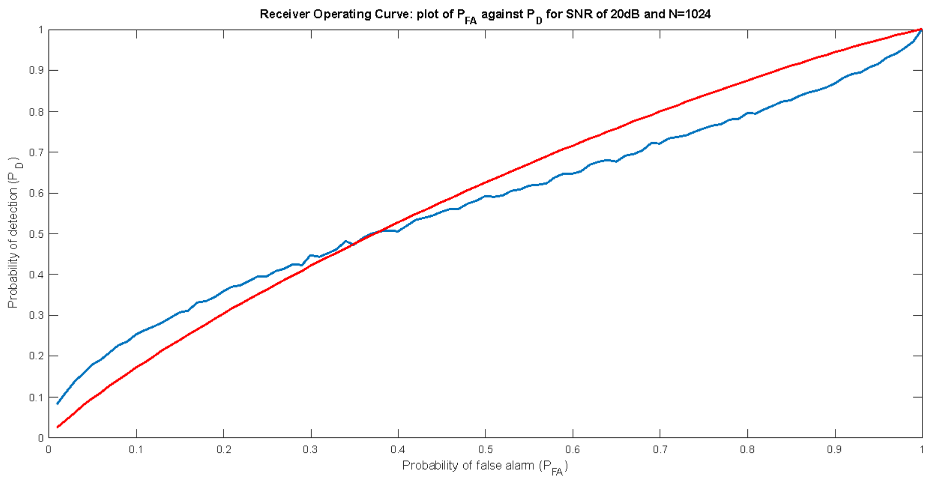

Figure 3: Receiver Operating Characteristics for SNR of -20dB

In the design of the energy detector, performance was analysed in accordance with IEEE 802.22 WRAN standard in terms of the probabilities of detection and false alarm as well as the SNR wall of the detector. The probabilities and SNR wall have a relationship with the number of samples (or sensing time).

Before discussing the results, there are some methods used to reiterate as steps to determining the design parameters for certain desired outputs (as specified by IEEE 802.22). First, the determination of the number of samples and compromise taken when processing power was constrained then, calculated minimum samples for a -20 dB energy detector.

The energy detector designed used the values of probabilities P_D and P_{FA} as 0.01 and 0.99 respectively for an SNR wall of -20dB. This resulted in a minimum number of samples as 218635.1766. The computation was done using Matlab with the formula in equation (22) where the inverse Q function is given by the formula $\sqrt{2}(1-2y)$ where erf^{-1} is the inverse error function given as $(z) = \sum_{k=0}^{\infty} \frac{c_k}{2k+1}\left(\frac{\sqrt{\pi}}{2}z\right)^{2k+1}$ so with the necessary parameters, $P_{fa} = 0.01, P_d = 0.99, SNR = 20dB$ the value of minimum number of samples is calculated (∞ can be replaced by a very large number).

This is a large number considering the next power of 2 that fits is 2^{18} and taking the amount of time to measure when done for 10000 Monte Carlo iterations for a single P_{FA} value (but there are 100 values).

If used for a real time signal (random signal) at a sampling rate of maybe 1MS/s then the sensing time will be $\frac{N}{samplerate} = 0.262144\ seconds$, which is not so much a requirement for an energy detector of $P_{FA} = 0.01$ and $P_D = 0.99$.

As $-20dB$ is the requirement of the energy detector for the spectrum sensing in a cognitive radio design, making the design no exception and the result of the probability of detection against the probability of false alarm is given in Figure 4, which shows that the design is recommendable at N=262144.

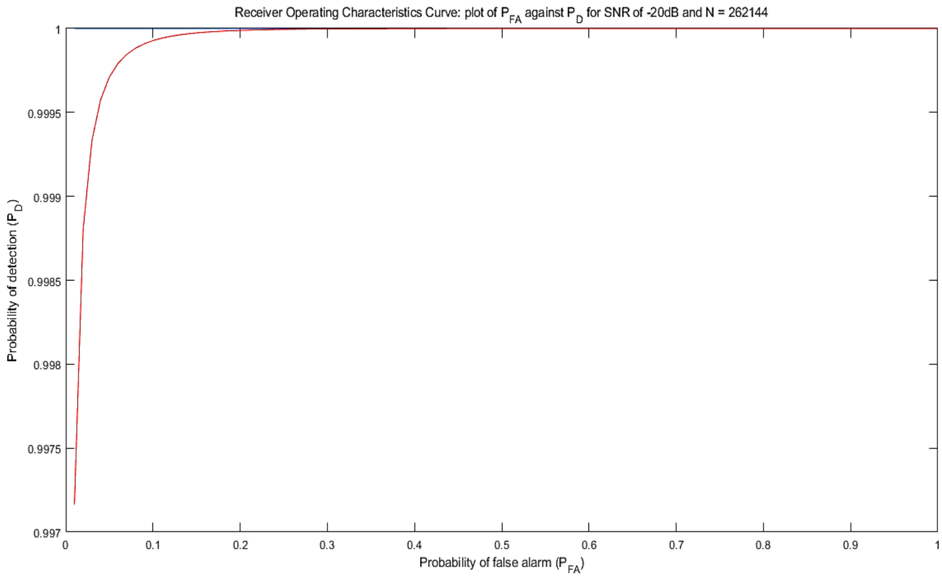

Figure 4: Receiver Operating Characteristics for a -20dB SNR wall energy detector

Figure 4, shows a perfect detection meeting the requirements of the probabilities (note that these probabilities are not tied to each other, P_D can be 0.1 and P_{FA} can be 0.95 or 0.99). This curve is known as the receiver operating characteristics (ROC) and it is used to check the accuracy of the design.

The closer the curve tends to the left-hand corner, the better the design. Due to the processing time it takes the PC, the design was done for; -10dB energy detector which only requires 2376.3034 samples whose next power of two is 4096.

Figures 2 to 3, show when N is 1024, the energy detector -15dB and less than -15dB will perform terribly as shown in the ROC, while it performs relatively well for SNR at -10dB and more than -10dB. Analysis on this design also extends to that of the -20dB which will be the same by replacing the number of samples in the design code.

With the number of samples at 262144, observation was carried out on the ROC for different SNR values; -20dB, and 5dB respectively as shown in Figures 4 (ROC). The ROC plots in Figure 4, show a good alignment of the curves for the case of SNR values.

V. Conclusion

The research in this paper has shown that there was a high correlation between simulated and theoretical values of the ROC plots for different SNRs.

The higher the SNR value for the energy detector, the better the performance and the lower the value, the worse the performance of the energy detector.

This value turned out to be approximately 95% for the case of known noise power. Using a bit of heuristics to determine the noise floor which is fairly accurate but one other assumption used in the study was that the noise floor remains constant in the primary spectrum.

However, considering the 5dB SNR as shown in Figure 6, it clearly distinguishes between the heuristics H0 and H1. So, the expectation was that the detector would have 100% detection for any false alarm probability. This design shows that it can eliminate noise when the noise power is perfectly known.

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